Characterizing Antennas in the Time and Frequency Domains

Everett G. Farr, *Senior Member, IEEE*

***Abstract*—Our antenna definitions standard currently has no terms that describe antenna performance in the time domain, possibly due to the complexity of the equations. However, if we express the antenna equations using quantities related to the square root of power, we obtain simple expressions valid in both the frequency and time domains. This leads to a number of new terms that should be considered for inclusion in the next revision of the standard. Doing so would provide a common language for discussing antenna performance in the time domain. It also would also add phase information to common frequency domain terms, such as antenna gain and radar cross section.**

***Index Terms*—Antenna characterization, time domain, antenna equation, Generalized Antenna Scattering Matrix (GASM), antenna impulse response, antenna transfer function, Electromagnetic Interference (EMI).**

I. Introduction

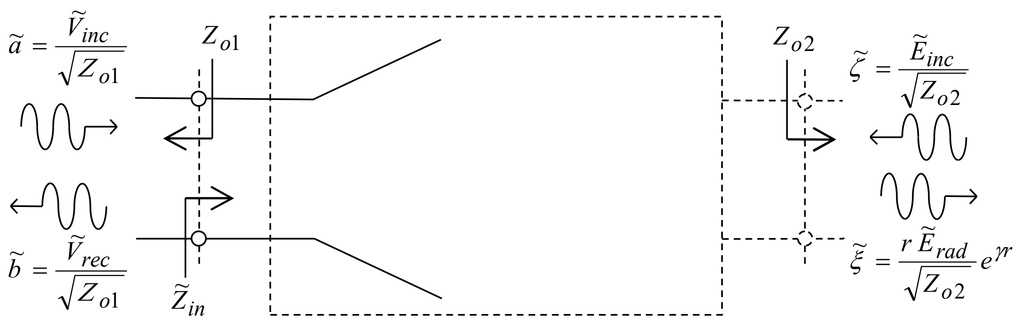
When describing antenna performance in the time domain, there is a surprising lack of standard terminology. The recently updated antenna definitions standard [1] treats only antenna performance in the frequency domain, so it is challenging to make comparisons between antennas in the time domain. To address this, we introduce an especially simple set of equations that clarifies how to extend existing antenna terminology into the time domain. In the process, we also show how to add phase information to common frequency domain terms, such as antenna gain and radar cross section.

This formulation applies to all linear antennas embedded in any lossless medium, including antennas with waveguide feeds. In this paper, we summarize the results; the complete derivation is provided in [2-3]. The derivation follows that of [5], and portions of the results are similar to those in [6, 7]. In this paper, we extend our earlier work in Sections VII and VIII.

II. The Antenna Equation

We consider first the case of far-field antenna performance, looking only at dominant polarization on boresight–we treat the general case later. The fields and waves are expressed using a two-port network formulation, as shown in Fig. 1, where Port 2 is a radiation port. The inputs and outputs are

$$
\begin{aligned}
\tilde{a} &= \frac{\tilde{V}_{inc}}{\sqrt{Z_{o1}}} && = \text{incident power wave} \\
\tilde{b} &= \frac{\tilde{V}_{rec}}{\sqrt{Z_{o1}}} && = \text{received power wave} \\
\tilde{\zeta} &= \frac{\tilde{E}_{inc}}{\sqrt{Z_{o2}}} && = \text{incident power flux density wave} \\
\tilde{\xi} &= \frac{r\,\tilde{E}_{rad}}{\sqrt{Z_{o2}}} e^{\gamma r} && = \text{radiated radiation intensity wave}
\end{aligned}
\tag{1}
$$



$$
\begin{bmatrix} \tilde{b} \\ \tilde{\xi} \end{bmatrix} = \begin{bmatrix} \tilde{\Gamma} & \tilde{h} \\ s\,\tilde{h}/(2\pi v) & \tilde{\ell} \end{bmatrix} \begin{bmatrix} \tilde{a} \\ \tilde{\zeta} \end{bmatrix}
$$

Fig. 1. The two-port network representing an antenna, on boresight, for dominant polarization.

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E. G. Farr is currently with Farr Fields, LC, Albuquerque NM, 87123, email: egfarr@gmail.com.

where $\gamma = s/v = jk$, $s = j\omega$, $k = \omega/v = 2\pi f/v$ is the propagation constant in the surrounding medium, and v is the velocity of propagation in the medium. Furthermore, Z_{o1} is the real reference impedance of the input port (often 50 Ω), Z_{o2} is the real impedance of the surrounding medium (often 120π Ω), and $\tilde{Z}_{in}$ is the complex impedance looking into the antenna. Note that the theory is extended to waveguide feeds below.

In addition, $\tilde{E}_{inc}$ is the incident electric field at the antenna, and $\tilde{E}_{rad}$ is the radiated electric far field. The tilde indicates a Laplace transform, to distinguish between frequency domain and time domain variables. Variables with tildes may be complex, and they may vary with frequency.

Real reference impedances, Z_{o1} and Z_{o2}, have been used without a loss of generality. A more general approach with complex reference impedances may be possible following the guidelines of Kurokawa [8] and others [9-11], but this is unnecessarily complex for the task at hand.[1] If the antenna is fed by a waveguide, the analogue of real reference impedances is lossless reference waveguide modes.

With these quantities defined, we now relate them to each other with the antenna equation,

$$\begin{bmatrix} \tilde{b} \\ \tilde{\xi} \end{bmatrix} = \begin{bmatrix} \tilde{\Gamma} & \tilde{h} \\ s\tilde{h}/(2\pi v) & \tilde{\ell} \end{bmatrix} \begin{bmatrix} \tilde{a} \\ \tilde{\zeta} \end{bmatrix}, \tag{2}$$

where $\tilde{\Gamma}$ is the reflection coefficient, $\tilde{h}$ is the antenna transfer function, and $\tilde{\ell}$ is the scattering coefficient. In the time domain, these become the reflection impulse response, the antenna impulse response, and the scattering impulse response, respectively. The matrix, referred to as the Generalized Antenna Scattering Matrix (GASM), is a complete description of antenna performance. An important feature of this model is that the inputs and outputs can all be measured easily in the time domain.

The four "power wave" quantities are related to well-known "power" quantities as

$$\begin{aligned} P_{inc}(s) &= ½\left|\tilde{a}\right|^2 & S(s) &= ½\left|\tilde{\zeta}\right|^2 \\ P_{rec}(s) &= ½\left|\tilde{b}\right|^2 & U(s) &= ½\left|\tilde{\xi}\right|^2 \end{aligned}, \tag{3}$$

where $P_{inc}(s)$ is the incident power at Port 1, $P_{rec}(s)$ is the received power at Port 1, $S(s)$ is the incident power flux density, and $U(s)$ is the radiated radiation intensity. All "power" quantities are average values, valid for $s = j\omega$. In all cases, the "power wave" quantities contain twice as much information as the "power" quantities, because they include phase.

III. Relationship to Previously Defined Quantities

The new quantities defined above are closely related to all the well-known antenna parameters. Thus, realized gain is related to antenna transfer function, $\tilde{h}$, as

$$G_r(s) = \frac{4\pi}{\lambda^2}\left|\tilde{h}\right|^2, \tag{4}$$

where λ is the wavelength in the medium. Note that $\tilde{h}$ has twice as much information as realized gain, because it includes phase. This is how one adds phase to realized gain. Note also that there is no need to add an extra factor to account for the resistive loss of the antenna, since this is already included in $\tilde{h}$. The usual relationship between gain and realized gain still holds,

$$G(s) = \frac{G_r(s)}{1-\left|\tilde{\Gamma}\right|^2}, \quad \tilde{\Gamma} = \frac{\tilde{Z}_{in} - Z_{o1}}{\tilde{Z}_{in} + Z_{o1}}, \tag{5}$$

where $G(s)$ is antenna gain. Curiously, the reflection coefficient, $\tilde{\Gamma}$, is not included in the antenna definitions standard [1], although the impedance mismatch factor, $1-|\tilde{\Gamma}|^2$, is included. We now need both.

The effective area or effective aperture, $A_e(s)$, is related to the antenna transfer function by

$$A_e(s) = \frac{\left|\tilde{h}\right|^2}{1-\left|\tilde{\Gamma}\right|^2}. \tag{6}$$

Effective length, $\tilde{h}_V$, is the ratio of the open circuit voltage to the incident electric field. It is related to $\tilde{h}$ by

$$\tilde{h}_V = \frac{\tilde{V}_{oc}}{\tilde{E}_{inc}} = \frac{\tilde{Z}_{in} + Z_{o1}}{\sqrt{Z_{o1} Z_{o2}}}\,\tilde{h}. \tag{7}$$

Finally, the radar cross section of the antenna when terminated in Z_{o1}, $\sigma(s)$, is related to the scattering coefficient, $\tilde{\ell}$, by

$$\sigma(s) = 4\pi\left|\tilde{\ell}\right|^2. \tag{8}$$

In all cases, the new parameters have twice as much information as the established parameters, because they include phase. This is necessary when transforming the equations into the time domain.

IV. Extension to the Time Domain, to two Polarizations, and to Arbitrary Angles

We convert the antenna equation (2) to the time domain by taking its inverse Laplace transform. Thus, we have

$$\begin{bmatrix} b(t) \\ \xi(t) \end{bmatrix} = \begin{bmatrix} \Gamma(t) & h(t) \\ h'(t)/2\pi v & \ell(t) \end{bmatrix} \stackrel{*}{\bullet} \begin{bmatrix} a(t) \\ \zeta(t) \end{bmatrix}, \tag{9}$$

where " ′ " indicates a time derivative, " $\stackrel{*}{\bullet}$ " is a matrix-product convolution operator defined as

$$\begin{bmatrix} s_{11}(t) & s_{12}(t) \\ s_{21}(t) & s_{22}(t) \end{bmatrix} \stackrel{*}{\bullet} \begin{bmatrix} a_1(t) \\ a_2(t) \end{bmatrix} = \begin{bmatrix} s_{11}(t)*a_1(t) + s_{12}(t)*a_2(t) \\ s_{21}(t)*a_1(t) + s_{22}(t)*a_2(t) \end{bmatrix} \tag{10}$$

and " * " is the convolution operator. Note that all the functions of time in (9) are real and causal, including $h(t)$, $\ell(t)$, $\Gamma(t)$, $a(t)$, $b(t)$, $\zeta(t)$, and $\xi(t)$.

Next we extend the antenna equation (2) to the general case

[1] If it becomes necessary to make a distinction between Kurokawa's more general concept of power waves, with complex reference impedances, and our simpler version, with real reference impedances, we propose using "lossless power wave" to describe our simpler version. To be consistent, one would also use "lossless power flux density wave" and "lossless radiation intensity wave."

of two polarizations, with arbitrary angles of incidence and observation. The extra polarization changes the 2-port network into a 3-port network. We use a spherical coordinate system of (r, θ, ϕ) with $\theta = 0$ on boresight. Thus, we have

$$\begin{bmatrix} \tilde{b}(\theta',\phi') \\ \tilde{\xi}_\theta(\theta,\phi,\theta',\phi') \\ \tilde{\xi}_\phi(\theta,\phi,\theta',\phi') \end{bmatrix} = \begin{bmatrix} \tilde{\Gamma} & \tilde{h}_\theta(\theta',\phi') & \tilde{h}_\phi(\theta',\phi') \\ s\tilde{h}_\theta(\theta,\phi)/(2\pi v) & \tilde{\ell}_{\theta\theta}(\theta,\phi,\theta',\phi') & \tilde{\ell}_{\theta\phi}(\theta,\phi,\theta',\phi') \\ s\tilde{h}_\phi(\theta,\phi)/(2\pi v) & \tilde{\ell}_{\phi\theta}(\theta,\phi,\theta',\phi') & \tilde{\ell}_{\phi\phi}(\theta,\phi,\theta',\phi') \end{bmatrix} \begin{bmatrix} \tilde{a} \\ \tilde{\zeta}_\theta(\theta',\phi') \\ \tilde{\zeta}_\phi(\theta',\phi') \end{bmatrix}, \tag{11}$$

where unprimed angles are angles of observation, and primed angles are angles of incidence. This may be expressed more compactly in vector notation as

$$\begin{bmatrix} \tilde{b} \\ \tilde{\boldsymbol{\xi}} \end{bmatrix} = \begin{bmatrix} \tilde{\Gamma} & \tilde{\boldsymbol{h}} \\ s\tilde{\boldsymbol{h}}/(2\pi v) & \tilde{\overline{\overline{\ell}}} \end{bmatrix} \cdot \begin{bmatrix} \tilde{a} \\ \tilde{\boldsymbol{\zeta}} \end{bmatrix}, \tag{12}$$

where a bold character indicates a vector, and a double line indicates a dyadic. The conversion of (11) or (12) to the time domain is straightforward, using the matrix product convolution operator defined in (10).

V. Signal Flow Graphs

Signal flow graphs (SFGs) can be used to visualize and simplify more complicated antenna problems. The SFG of the antenna equation (2) is shown in Fig. 2. This could be used, for example, to calculate the radiated field when the antenna is driven by a source of arbitrary complex impedance. It could also be used to calculate the scattered field when the antenna is loaded with an arbitrary complex impedance. The SFG for the complete antenna equation (10) is provided in [2-4], along with the solution to a variety of sample problems.

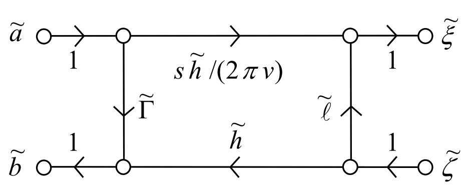

Fig. 2. Signal flow graph of the antenna equation, on boresight, for dominant polarization.

As an example, we use the above SFG to calculate the field radiated from a source of arbitrary impedance, on boresight, for dominant polarization. The goal is to find the ratio of $\tilde{\xi}$ to $\tilde{b}_s$, where $\tilde{b}_s = \tilde{a}$ is the power wave generated by the source. A sketch of the SFG with source added is shown in Fig. 3. Using Mason's rule, the graph resolves as

$$\tilde{\xi} = \frac{1}{1 - \tilde{\Gamma}\tilde{\Gamma}_s} \frac{s\tilde{h}}{2\pi v} \tilde{b}_s . \tag{13}$$

This example shows that the antenna equation (2) is general for arbitrary complex source impedance, load impedance, and antenna input impedance.

To find the source parameters, $\tilde{\Gamma}_s$ and $\tilde{b}_s$, we need to establish a relationship between a power wave source and a Thévenin equivalent source. The result is shown in Fig. 4.

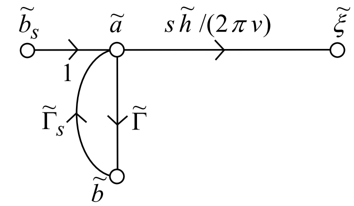

Fig. 3. Signal flow graph for radiation from a source of arbitrary impedance.

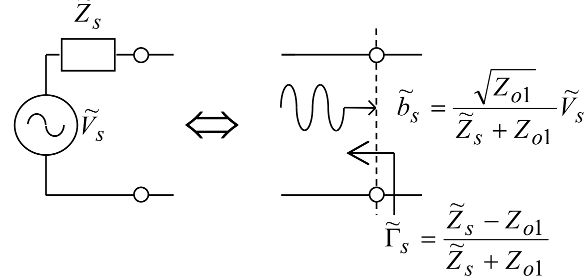

Fig. 4. The relationship between a Thévenin equivalent source (left) and a power wave source (right) at a port with reference impedance Z_{o1}.

From (13), we see it is not necessary to know explicitly the reference impedance, Z_{o1}, if one can find the two reflection coefficients, $\tilde{\Gamma}_s$ and $\tilde{\Gamma}$. So (13) can be used with waveguide feeds, for which reference impedances are ill-defined, but reflection coefficients are easily available.

VI. An Example Antenna Impulse Response

To illustrate the antenna impulse response concept, we consider an example UWB antenna, the Farr Fields model IRA-3Q, shown in Fig. 5. This is an Impulse Radiating Antenna with a diameter of 46 cm, and a frequency range of 250 MHz to 20 GHz. Its antenna impulse response on boresight, $h(t)$, is shown in Fig. 6, demonstrating its nondispersive properties. The Full-Width Half Max of the impulsive portion of its antenna impulse response is 38 ps. The reference impedance, Z_{o1}, at the input port is 50 Ω, which is also the approximate input impedance of the antenna. In Figs. 7-8, we show this antenna's boresight gain and reflection coefficient, demonstrating its impressive performance over nearly two decades of bandwidth.

Fig. 5. The Farr Fields model IRA-3Q.

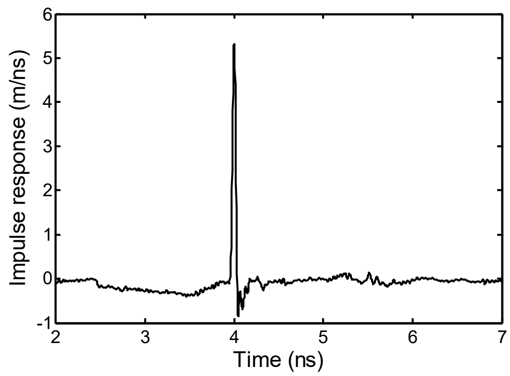

Fig. 6. Antenna impulse response on boresight, $h(t)$, of the Farr Fields model IRA-3Q.

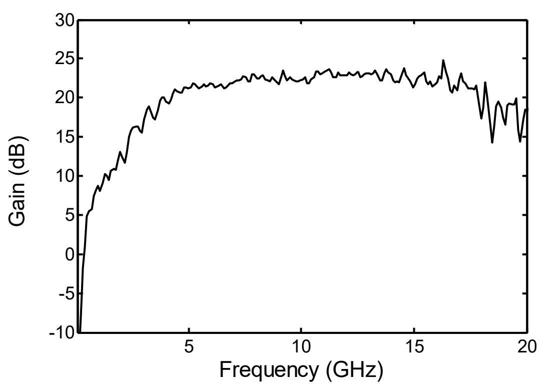

Fig. 7. Antenna gain on boresight, $G(f)$, of the Farr Fields model IRA-3Q.

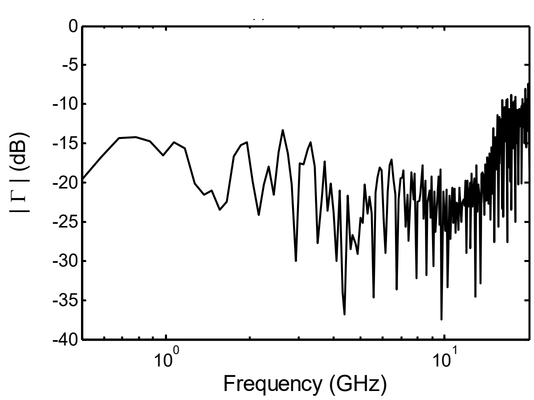

Fig. 8. Reflection coefficient magnitude, $|\Gamma(f)|$, of the Farr Fields model IRA-3Q.

VII. Application to Electromagnetic Interference

The antenna equation (2) has an important application in the field of Electromagnetic Interference (EMI). Consider the problem shown in Fig. 9, which shows a port located inside an imperfectly shielded enclosure or electronics cabinet. We need to describe both the fields radiated out of the enclosure from the port, and the voltages coupled into the port from exterior fields. This has to be done in a way that is meaningful in both transmission and reception, in both the frequency and time domains. This was always a challenging problem, however, we can think of this as an unintentional antenna. In that case, the antenna transfer function and antenna impulse response provide the most natural method of characterizing radiation and reception.

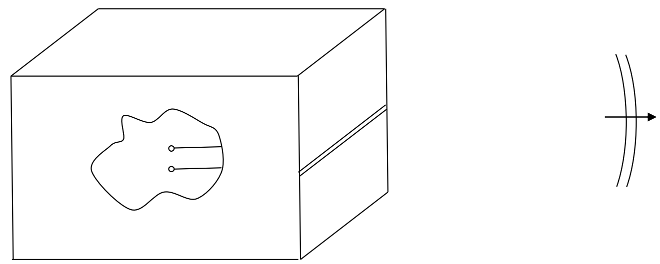

Fig. 9. An imperfectly shielded enclosure with an interior port.

VIII. New Definitions

Next, we consider new definitions that emerge from the antenna equation (2). Many people are surprised to learn that there are almost no equations in our antenna definitions standard [1]; all definitions are expressed in words. At first, this might seem unnecessary, but it has the benefit of forcing us to use the simplest possible expressions in our definitions. Because the antenna equation (2) is simple enough to put into words, it seems like the right description of antenna performance. Two sample terms from (2) might be drafted as follows:

antenna transfer function (in a given direction): in reception, the ratio of the received power wave to the incident power flux density wave. In transmission, the ratio of the radiated radiation intensity wave to the time derivative of the source power wave, multiplied by 2π times the propagation velocity of the medium.

antenna impulse response (in a given direction): the inverse Laplace transform of the antenna transfer function.

These are not complete, because they ignore polarization. But that will be handled in the same way it is handled in the definition of gain. Similar definitions can be provided for reflection coefficient, reflection impulse response, scattering coefficient, and scattering impulse response.

IX. Conclusions

All of the standard quantities used to describe antenna performance in the frequency domain are scalar, however, complex quantities will be needed to get to the time domain.

The antenna equation (2) is the simplest expression of antenna response we have found that includes all phase information, works in both the time and frequency domains, and describes performance in both transmission and reception. It is also the only formulation we have found that is simple enough to be expressed in words. We encourage the use of the new parameters found in the antenna equation as a first step toward establishing new standard terms.

Much more information is available in [2-4]. In [2,3], we derive all the results shown here. We also solve a number of additional sample problems with signal flow graphs, and we discuss mutual coupling in an antenna array, transient antenna patterns, and many additional antenna terms. There is also a long reference list of prior work. In [4], we calculate antenna parameters for some electrically small antennas that can be described in closed form. We also show how to add a matching circuit to an antenna, as an example of the use of signal flow graphs.

Acknowledgment

I wish to thank Mr. Leland H. Bowen for building and testing the antenna shown in Section VI.

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Note: Sensor and Simulation Notes are available from the Summa Foundation website, http://ece-research.unm.edu/summa//notes/index.html .

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Author Information

Everett G. Farr (egfarr@gmail.com) is the owner of Farr Fields, LC (formerly Farr Research, Inc.), in Albuquerque, NM, USA. He specializes in the development and measurement of UWB antennas. He is a co-recipient of the 2006 John Kraus Antenna Award, and he is the current editor of the Summa Foundation Note Series.