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Time Domain Characterization of Antennas with TEM Feeds

Everett G. Farr
Farr Research, Inc.

Carl E. Baum
Air Force Research Laboratory / DEHP

Abstract

We provide here a clear accounting of how to relate the source voltage from a 50 Ω source, through a transmitting and receiving antenna, into a 50 Ω load. Transmission coefficients at impedance mismatches are handled explicitly. A relationship is provided between the area under the antenna's impulse response and the line dipole moment of the aperture.

We also show that the antenna equations are simplified considerably by normalizing voltages and electric fields to the local characteristic impedance. In doing so, we introduce a universal time domain antenna impulse response, $h_N(t)$. When the antenna equations are expressed in terms of $h_N(t)$, the equations take on a number of useful simplifications. First, the transmission coefficients between the feed cable and antenna impedance are eliminated. Second, the impedance of the antenna is eliminated. Normally an assumption has to be made that the antenna impedance is a constant, but it does not appear in our formulation. Third, the normalized impulse response applies to both transmission and reception equations. Fourth, the expressions in transmission and reception are as simple as one can imagine that they will ever get. Finally, by writing our equations in this manner, we are able to tie the theory back to measurements that are routinely made on transient antennas. The process of normalizing voltages to the local characteristic impedance is reminiscent of S-parameters in circuit theory.

I. Introduction

In this note we introduce certain simplifications into the time domain antenna equations. In doing so, we normalize all voltages and electric fields to local characteristic impedances. After doing so, we introduce the normalized impulse response, $h_N(t)$, which completely describes the behavior of antennas with TEM feeds in both transmission and reception. By introducing this new quantity, there is no need to specify the antenna impedances or transmission coefficients that are normally needed. This expression of the antenna equations adds simplicity and clarity to all the familiar antenna equations. In addition, calibration procedures for antennas are greatly simplified.

We begin by clarifying some of the notation that is used in conventional antenna equations. In doing so, we help to explain how to take into account certain transmission coefficients in the signal processing used to extract the impulse response of the antenna, $h(t)$. We also show how the convolution with an impulse-like antenna characteristic can be approximated by a multiplication by the area under the impulse. In addition, we show the correct normalization of the impulse response, so the area under the impulse can be related to the effective height of the line charge in the aperture plane.

Finally, we show how to simplify the conventional equations by normalizing the equations to the local characteristic impedance. The process is reminiscent of S-parameters in microwave circuit theory. In doing so, not only are the transmission coefficients eliminated, but the antenna feed impedance is also eliminated. This leads naturally to a simple calibration procedure using two identical antennas. Furthermore, we can relate the new $h_N(t)$ to the equivalent height of the charge distribution projected in the aperture plane. This can be used as a simple check on the results of the calibration.

II. Radiated Field, Conventional Expression

We consider here the conventional expressions for radiated field, and in the two succeeding sections we consider the received voltage and the combined (transmit and receive) expressions. Later, we essentially repeat the process with the normalized expressions. The configuration for both transmit and receive antennas are shown in Figure 2.1.

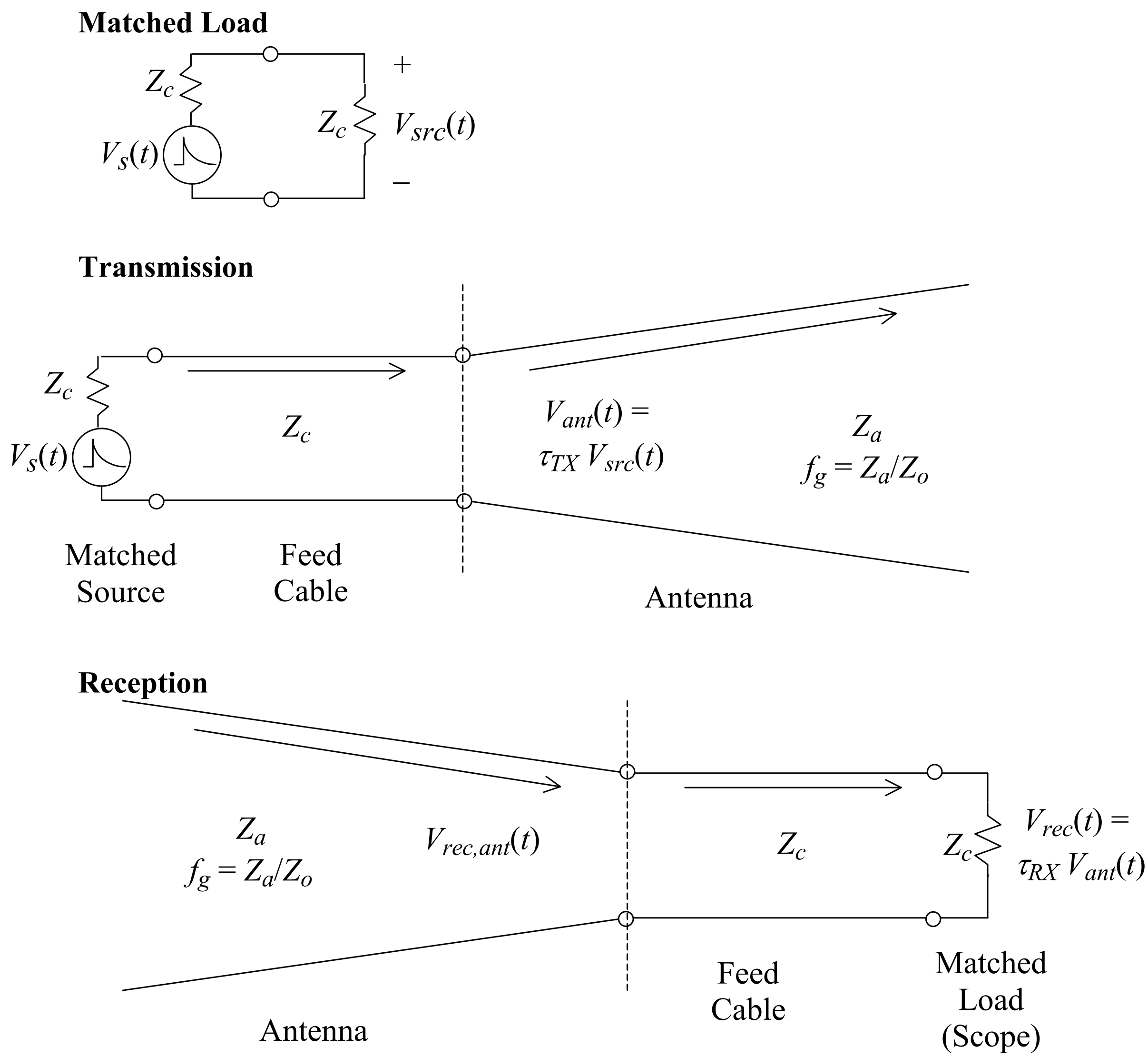

Figure 2.1. The antenna and feed cable in transmission and reception

We have introduced certain simplifications into our equations to enhance clarity. First, we have chosen intentionally to consider only the antenna performance on boresight for dominant polarization. The extension to the more general case that includes angle and polarization dependence is trivial, and does not affect in any way the choice of normalization. We show an example of how this is accomplished in Section 9 of this paper.

We begin by describing the transmission equations. For a transmitting antenna, the radiated field is related to the source voltage by [1]

$$
\begin{aligned}
E_{rad}(t) &= \frac{1}{2\pi\, r\, c\, f_{g,TX}} h_{TX}(t) \circ \frac{dV_{ant,TX}(t)}{dt} \\
f_{g,TX} &= \frac{\text{Impedance on TEM feed of TX Antenna}}{377\,\Omega} = \frac{Z_{a,TX}}{Z_o} \\
\frac{dV_{ant,TX}(t)}{dt} &= \tau_{TX} \frac{dV_{src}(t)}{dt} \\
\tau_{TX} &= \frac{2 Z_{a,TX}}{Z_{a,TX} + Z_c}\,, \qquad Z_c = 50\,\Omega
\end{aligned}
\tag{2.1}
$$

Note that $V_{ant,TX}(t)$ is the voltage excited on the TEM feed structure of the transmitting antenna, and τ_{TX} is the voltage transmission coefficient from the 50 Ω feed cable to the antenna. In addition, "$\circ$" indicates convolution. Furthermore, $V_{src}(t)$ is the voltage one would measure into an oscilloscope with a 50 Ω input impedance. We assume here that $Z_{a,TX}$ is a real constant. This will be generalized later to allow time (or frequency) dependence.

The transmission coefficient in the above equation, τ_{TX}, is the voltage transmission coefficient the wave experiences in going from the 50 Ω feed cable to the antenna. We have assumed here a cable impedance of $Z_c = 50$ Ω, but this is trivial to adjust for other cable impedances.

When there is a balun in the circuit, special care must be used. The above equations apply, but a different form of the transmission coefficient must be used. For example, there is a balun that converts a 50 Ω impedance to a 200 Ω impedance, using two sections of 100 Ω transmission line connected in series at one end and in parallel at the other end. In this case, the voltage doubles, so the transmission coefficient is two. If one simply used the expression for transmission coefficient shown in (2.1), one would obtain an incorrect result.

Note that there are an infinite number of ways to define the impulse response, $h(t)$, depending on the assumed load. In [1], Baum treats cases with open circuits, $h_V(t)$, short circuits, $h_I(t)$, and loads matched to the transmission line impedance, $h_w(t)$ or $h_t(t)$. All of these are valid methods of expressing the antenna equations. For the purposes of this note, we choose to talk only about a specific impulse response, $h(t)$, which refers to the condition where the antenna is terminated in a real impedance equal to the impedance of the TEM feed structure. This is a particularly simple form to use, because the integral over the impulsive portion of $h(t)$ is related to the effective height of the charge distribution projected into the aperture plane, h_{eq}, as we will see shortly.

By combining the above equations, we have

$$\boxed{E_{rad}(t) = \frac{\tau_{TX}}{2\pi r c f_{g,TX}} h_{TX}(t) \circ \frac{dV_{src}(t)}{dt}} \tag{2.2}$$

This is the complete result for the transmitting antenna.

Now we have a possible simplification available to us, because the convolution with an impulse-like function can be approximated by a multiplication by the area under the impulse. Thus [2],

$$\begin{aligned} E_{rad}(t) &\approx \frac{\tau_{TX}\ h_{a,TX}}{2\pi r c f_{g,TX}} \frac{dV_{src}(t)}{dt} \\ h_{a,TX} &= \int_{\text{Impulse}} h_{TX}(t)\, dt \end{aligned} \tag{2.3}$$

This integral is just half the effective height of the line charge distribution when projected into the aperture plane. So it is equal to half the plate separation in a TEM horn, $a/(2\sqrt{2})$ in a half IRA with two arms, or $a/\sqrt{2}$ in a full IRA with four arms. In other words,

$$\begin{aligned} h_{a,TX} &= \begin{cases} h_{eq}/2 \\ h/2 \quad \text{in half or whole TEM horn} \\ a/\sqrt{2} \quad \text{in Full Reflector IRA (4 arms)} \\ a/(2\sqrt{2}) \quad \text{Half Reflector IRA (2 arms)} \end{cases} \\ h_{eq} &= \text{dipole moment / charge in projected aperture} \\ h &= \text{plate separation in full TEM horn} \\ h &= \text{plate-to-ground plane distance in half TEM horn} \\ a &= \text{radius in full or half IRA} \end{aligned} \tag{2.4}$$

Alternatively, we could use the expression, from [2]

$$h_{a,TX} = -\frac{1}{\Delta v} \oint_{C_a} v(\zeta)\, d\zeta \tag{2.5}$$

where $v(\zeta)$ is the imaginary part of the complex potential that describes the aperture fields, and C_a is a contour around the aperture.

Now we see the importance of choosing the correct load impedance when defining $h(t)$. It is only when the load impedance is equal to Z_a that the above integrals apply. And when making measurements, it is necessary to check the results by comparing the area of the measured impulse response to the results above.

III. Received Voltage, Conventional Expression

Consider now the case of a receiving antenna, with a plane-wave incident field. The received voltage on the antenna is related to the incident field by [1]

$$
\begin{aligned}
V_{rec,ant}(t) &= h_{RX}(t) \circ E_{inc}(t) \\
V_{rec}(t) &= \tau_{RX} V_{rec,ant}(t) \\
\tau_{RX} &= \frac{2\,Z_c}{Z_c + Z_{a,RX}} \,, \qquad Z_c = 50\,\Omega
\end{aligned}
\tag{3.1}
$$

Note that $V_{rec,ant}(t)$ is the received voltage on the antenna, and $V_{rec}(t)$ is the received voltage onto a 50 Ω feed cable. An oscilloscope will measure $V_{rec}(t)$. The transmission coefficient relates the received voltage to the voltage on the antenna. Note also that the subscript "RX" refers only to whether a given antenna is being used at the moment for transmission or reception. For any antenna, the equations are defined such that $h_{RX}(t) \equiv h_{TX}(t)$. We make the distinction only because in a two-antenna problem there may be two different antennas being used. Finally, note that once again, for the purposes of defining $h(t)$, we assume that the antenna is terminated in the real impedance of the TEM feed.

Once again, care must be used in defining the transmission coefficient when a balun is used. With the standard balun matching a 200 Ω antenna to a 50 Ω cable, the voltage transmission coefficient is 1/2.

Combining the above equations, we find the total received voltage is

$$
\boxed{V_{rec}(t) = \tau_{RX}\; h_{RX}(t) \circ E_{inc}(t)}
\tag{3.2}
$$

This is the complete equation for receive mode. As before, the convolution can be approximated by a multiplication by the area under the impulse. Thus,

$$
\begin{aligned}
V_{rec}(t) &\approx \tau_{RX}\; h_{a,RX}\, E_{inc}(t) \\
h_{a,RX} &= \int_{\text{Impulse}} h_{RX}(t)\, dt
\end{aligned}
\tag{3.3}
$$

We can use the same areas give previously for the transmission case in (2.4).

It is convenient to define an effective height that expresses a simple proportionality between the incident field and the voltage into a scope. Thus, we have

$$
\begin{aligned}
V_{rec}(t) &\approx h_{eff}\; E_{inc}(t) \\
h_{eff} &= \tau_{RX} h_{a,RX}
\end{aligned}
\tag{3.3}
$$

This simple proportionality may be the expression most commonly used, but it is only valid when the FWHM of $h_{RX}(t)$ is much less than the measured signal.

IV. Combined Equation, Conventional Expression

We can now find a relationship between the source and received voltages in a two-antenna system. We do so by using the radiated field expression, (2.2) as the incident field in the received voltage expression, (3.2). After combining the two equations, we have

$$\boxed{V_{rec}(t) \;=\; \frac{\tau_{RX}\,\tau_{TX}}{2\pi\, r\, c\, f_{g,TX}}\, h_{RX}(t)\circ h_{TX}(t)\circ \frac{dV_{src}(t)}{dt}} \tag{4.1}$$

Again, we can approximate the convolutions as multiplications by a scalar, so we have

$$V_{rec}(t) \;\approx\; \frac{\tau_{RX}\,\tau_{TX}\, h_{a,RX}\, h_{a,TX}}{2\pi\, r\, c\, f_{g,TX}}\, \frac{dV_{src}(t)}{dt} \tag{4.2}$$

This completes the development.

When calibrating an antenna, one normally measures the response from two identical antennas. Since in this case $h_{RX}(t) \equiv h_{TX}(t)$, equation (4.1) becomes

$$V_{rec}(t) \;=\; \frac{\tau_{RX}\,\tau_{TX}}{2\pi\, r\, c\, f_{g,TX}}\, h(t)\circ h(t)\circ \frac{dV_{src}(t)}{dt} \tag{4.3}$$

To extract $h(t)$, one has to convert to the frequency domain as

$$\boxed{\tilde{h}(\omega) \;=\; \sqrt{\frac{2\pi\, r\, c\, f_{g,TX}\;\; \tilde{V}_{rec}(\omega)}{\tau_{TX}\;\tau_{RX}\;\; j\omega\;\; \tilde{V}_{src}(\omega)}}} \tag{4.4}$$

Both the source and receive voltages are measured directly. The transmission coefficients and antenna impedance, f_g, are assumed to be known scalars. A similar technique was also used in [3, eqn.(3.5)] and [4, Section 2]. Note that before the complex square root is taken, the phase has to be unwrapped, in order to avoid nonphysical results.

But the above calibration procedure has the disadvantage that one has to make assumptions about f_g and the two transmission coefficients. In particular, one has to assume that the antenna impedance is a constant, when in fact it can vary. It would be far preferable to have a set of antenna expressions that did not require this artificial assumption. The remainder of this paper will show how to eliminate the impedances and transmission coefficients from all the antenna equations.

V. The Problem With the Conventional Expressions

The above formulation of the conventional antenna equations has a problem. This formulation requires a knowledge of $f_{g,TX}$, the normalized impedance of the transmitting antenna, as well as two transmission coefficients. When building antennas, we normally strive to have the antenna input impedance be a constant. But this is a design goal that can be a challenge to achieve in practice. This can be verified by looking the TDR of any number of existing antennas. So the antenna impedance is normally just set to the real input impedance of the antenna at early time. This seems like a somewhat arbitrary approximation, which one should be able to avoid.

It turns out that there is a very simple way to avoid the above-mentioned artificial assumptions. All one needs to do is normalize the voltages and fields to the local characteristic impedance of the transmission line or medium. By doing so, we shall see that the antenna impedance drops out of all the antenna equations. All the expressions will then be consistent with features that are easily measurable. In addition, the equations become as simple as they can ever get.

To implement the new set of normalized equations, we follow a parallel development of the radiated fields, the received voltage, and the combined equation, analogous to Sections II through IV above.

VI. Radiated Field, Normalized Expressions

Let us now apply the normalization process to the radiated field expressions. Starting with (2.2), we normalize the fields and voltages to the local characteristic impedance, and expand $f_{g,TX}$. The normalization procedure is somewhat reminiscent of S-parameters in microwave circuit theory. Thus, we find

$$\begin{aligned} \frac{E_{rad}(t)}{\sqrt{Z_o}} &= \frac{1}{2\pi r c}\left[\frac{Z_o}{Z_{a,TX}}\,\tau_{TX}\,\frac{\sqrt{Z_c}}{\sqrt{Z_o}}\right] h_{TX}(t)\circ\frac{1}{\sqrt{Z_c}}\frac{dV_{src}(t)}{dt} \\ &= \frac{1}{2\pi r c}\left[\sqrt{\frac{Z_c}{Z_{a,TX}}}\,\tau_{TX}\right]\frac{h_{TX}(t)}{\sqrt{f_{g,TX}}}\circ\frac{1}{\sqrt{Z_c}}\frac{dV_{src}(t)}{dt} \end{aligned} \tag{6.1}$$

To simplify the above expression, we define a new transmission coefficient that relates the square root of the power launched onto the antenna to the incident power on the feed cable. We will refer to this as the square-root-power transmission coefficient, τ_p. and it is calculated as

$$\tau_{p,TX} = \frac{2\sqrt{Z_c\,Z_{a,TX}}}{Z_c + Z_{a,TX}} = \tau_{TX}\sqrt{\frac{Z_c}{Z_{a,TX}}} \tag{6.2}$$

Note that in the case of a perfect balun, $\tau_p = 1$. Substituting (6.2) into (6.1) gives

$$\frac{E_{rad}(t)}{\sqrt{Z_o}} = \frac{1}{2\pi r c}\,\frac{\tau_{p,TX}\;h_{TX}(t)}{\sqrt{f_{g,TX}}}\circ\frac{1}{\sqrt{Z_c}}\frac{dV_{src}(t)}{dt} \tag{6.3}$$

Let us now define a normalized impulse response, $h_N(t)$ as

$$h_{N,TX}(t) = \frac{\tau_{p,TX}}{\sqrt{f_{g,TX}}}\;h_{TX}(t) \tag{6.4}$$

Then we have a simplified expression for the radiated field as

$$\boxed{\frac{E_{rad}(t)}{\sqrt{Z_o}} = \frac{1}{2\pi r c}\;h_{N,TX}(t)\circ\frac{1}{\sqrt{Z_c}}\frac{dV_{src}(t)}{dt}} \tag{6.5}$$

This is the final result for transmission. Note that the antenna's transmission characteristic is described completely by its $h_N(t)$. Note also that no impedance or transmission coefficients are needed in the expression.

VII. Received Voltage, Normalized Expression

We now repeat the process with the equation for the received voltages. Starting with (3.2), we once again normalize the voltages and electric fields to the local characteristic impedance. This results in

$$\begin{aligned}\frac{V_{rec}(t)}{\sqrt{Z_c}} &= \sqrt{\frac{Z_o}{Z_c}}\;\tau_{RX}\;\; h_{RX}(t) \circ \frac{E_{inc}(t)}{\sqrt{Z_o}} \\ &= \left[\sqrt{\frac{Z_{a,RX}}{Z_c}}\;\tau_{RX}\right] \frac{h_{RX}(t)}{\sqrt{f_{g,RX}}} \circ \frac{E_{inc}(t)}{\sqrt{Z_o}}\end{aligned} \tag{7.1}$$

Once again, we invoke the square-root-power transmission coefficient, τ_p. , which is defined as

$$\tau_{p,RX} = \frac{2\sqrt{Z_c\; Z_{a,RX}}}{Z_c + Z_{a,RX}} = \tau_{RX}\sqrt{\frac{Z_{a,TX}}{Z_c}} \tag{7.2}$$

It is interesting to note the symmetry in the square-root-power transmission coefficient. Comparing (7.2) to (6.2), we find no difference in form between the transmitted and received cases. Thus, the discontinuity in square-root power at a transmission line discontinuity is the same, no matter which direction the wave comes from. And with a perfect balun, $\tau_p = 1$. Substituting (7.2) into (7.1) gives

$$\frac{V_{rec}(t)}{\sqrt{Z_c}} = \frac{\tau_{p,RX}\;\; h_{RX}(t)}{\sqrt{f_{g,RX}}} \circ \frac{E_{inc}(t)}{\sqrt{Z_o}} \tag{7.3}$$

Once again, we define a normalized impulse response as

$$h_{N,RX}(t) = \frac{\tau_{p,RX}}{\sqrt{f_{g,RX}}}\; h_{RX}(t) \tag{7.4}$$

Then the expression for the received voltage becomes

$$\boxed{\frac{V_{rec}(t)}{\sqrt{Z_c}} = h_{N,RX}(t) \circ \frac{E_{inc}(t)}{\sqrt{Z_o}}} \tag{7.5}$$

This is the final result for reception. The receive antenna is described completely by its $h_N(t)$, without any impedances or transmission coefficients.

We now arrive at the simple result that the normalization we want for reception in equation (7.4), is precisely the same as that which we want for transmission, in equation (6.4)! This is a fairly remarkable and unexpected result. **So to describe the antenna's performance in either transmission or reception, one need only know its $h_N(t)$.** One cannot imagine a simpler pair of equations than those in (6.5) and (7.5). There will be no need to specify the antenna impedance for either transmission or reception. There is also no need to calculate the two transmission coefficients needed in the conventional development.

VIII. Combined Equation, Normalized Expression

We can now find the combined equation by combining the radiated field of equation (6.3) with the received voltage of equation (7.3). Thus, we have

$$\boxed{V_{rec}(t) \;=\; \frac{1}{2\pi r c} h_{N,RX}(t) \circ h_{N,TX}(t) \circ \frac{dV_{src}(t)}{dt}} \tag{8.1}$$

This equation compares to equation (4.1), but with some important simplifications. There are no transmission coefficients, and there is now no normalized impedance of the antenna.

Once again, one might use the above equation to calibrate an antenna using two identical antennas. The equation for extracting the $h_N(t)$ is

$$\boxed{\tilde{h}_N(\omega) \;=\; \sqrt{\frac{2\pi r c \;\; \tilde{V}_{rec}(\omega)}{j\omega \;\; \tilde{V}_{src}(\omega)}}} \tag{8.2}$$

Upon comparing this equation to (4.4), we see that there are no transmission coefficients or antenna impedances to calculate. Note also that if there is a time dependence in any of the transmission coefficients or in the antenna impedance, Z_a, it is completely contained within the normalized impulse response, $h_N(t)$. So in this sense, this is the most natural method for calibrating antennas. Note that there is some ambiguity in the sign of $h_N(t)$ due to the square root, but one can resolve that from the physics of the antenna.

Note also that this formulation has used a scalar description for h_N, $h_{N,TX}$, and $h_{N,RX}$. Implicitly, this has assumed that the vector orientation of $\vec{h}_{N,RX}$ and $\vec{h}_{N,TX}$ is known a priori. For the special (but common) case that the antenna has a symmetry plane S which also contains $\vec{1}_i$, then the vector orientations are frequency (and time) independent, and either perpendicular or parallel to S. Furthermore, $\vec{h}_{N,RX}$ and $\vec{h}_{N,TX}$ can be taken as parallel by appropriate orientation of the two antennas with S as a common symmetry plane. Thereby, $\vec{h}_N$ is given the same orientation. This works well for both E- and H-plane symmetry planes (vertical and horizontal).

After measuring an antenna pattern it will always be necessary to compare the results to predictions. Once $h_N(t)$ is found, one can extract $h(t)$, with knowledge of τ_p and f_g. One can then integrate $h(t)$ and compare the integral to the expected values in (2.4). It is likely that both τ_p and f_g have some time dependence, but for the purposes of checking one's data, it is reasonable to assume that they are constant at early time.

Note also that we have eliminated the polarization and angle dependence of the antennas, but it is possible to add this later for completeness. We do so in the next section.

Finally, we note that it is possible to express these equations by an alternative method, and we do so in Appendix A.

IX. Polarization and Angle Dependence in the Normalized Equations

For completeness, we provide here an example of how to add polarization and angle dependence to all the previous equations in this paper. Thus, we do so here for the set of normalized equations.

For transmission, from (6.5) we have

$$\boxed{\frac{\vec{E}_{rad}(\theta,\phi,t)}{\sqrt{Z_o}} = \frac{1}{2\pi r c}\; \vec{h}_{N,TX}(\theta,\phi,t) \circ \frac{1}{\sqrt{Z_c}} \frac{dV_{src}(t)}{dt}} \tag{9.1}$$

where

$$\vec{h}_{N,TX}(\theta,\phi,t) = \frac{\tau_{p,TX}}{\sqrt{f_{g,TX}}}\; \vec{h}_{TX}(\theta,\phi,t) \tag{9.2}$$

For reception, from (7.5) we have

$$\boxed{\frac{V_{rec}(t)}{\sqrt{Z_{c1}}} = \vec{h}_{N,RX}(\theta,\phi,t) \;\overset{\circ}{\bullet}\; \frac{\vec{E}_{inc}(\theta,\phi,t)}{\sqrt{Z_o}}} \tag{9.3}$$

where the "$\overset{\circ}{\bullet}$" operator is a dot-product convolution. The combined equation, from (8.1), is

$$\boxed{V_{rec}(\theta,\phi,\theta',\phi',t) = \frac{1}{2\pi r c} \vec{h}_{N,RX}(\theta',\phi',t) \;\overset{\circ}{\bullet}\; \vec{h}_{N,TX}(\theta,\phi,t) \circ \frac{dV_{src}(t)}{dt}} \tag{9.4}$$

where the primed angles refer to the orientation of the receive antenna, and the unprimed angles refer to the orientation of the transmit antenna.

X. Discussion

While we have stated that these equations are specific to antennas with TEM feeds, there is no real reason to limit the scope of the equations in this way. In fact, we can use equation (8.2) to define the impulse response of any antenna, $h_N(t)$, in either the time or frequency domain. Since this is so closely tied to the measurements one makes in a laboratory, one might consider this a more general description of antenna performance than others that have been previously proposed. And by allowing (8.2) to define $h_N(t)$, we eliminate all ambiguity resulting from impedances and transmission coefficients that are not constant with time or frequency.

XI. Conclusion

We have simplified the set of equations describing the class of impulse antennas that are most commonly used. These equations eliminate the need to assume that certain impedances and transmission coefficients are constants. They are tied in directly to the laboratory measurements one commonly makes, so they provide a simple procedure for antenna calibration using two identical antennas. The simplicity of the method is illustrated by comparing the simplicity of equation (8.2) to the relatively more complex equation (4.4). The resulting normalized impulse response can be checked for validity by converting back to the unnormalized form, integrating the resulting waveform, and comparing to the equivalent height of the projected line dipole in the aperture plane.

Appendix A: An Alternative Formulation

We consider here an alternative formulation that is more closely tied in with [1]. The general antenna equations are as follows [1]:

$$\begin{aligned}
&\text{Transmission} && \tilde{\vec{E}}_f(\vec{r},s) = \frac{e^{-\gamma r}}{r}\,\tilde{\vec{F}}_t(\vec{1}_r,s)\,\tilde{V}_t(s) \\
&\text{Reception} && \tilde{V}_L(s) = \tilde{\vec{h}}_t(\vec{1}_i,s)\bullet\tilde{\vec{E}}_o(s) \\
&\text{Reciprocity} && \tilde{\vec{F}}_t(\vec{1}_r,s) = \frac{s\,Z_o}{2\pi\,c\,R}\,\tilde{\vec{h}}_t(-\vec{1}_r,s)
\end{aligned} \tag{A.1}$$

where the subscript "*t*" refers to voltages at the input to a transmission line, and V_L is the voltage across the load resistor. For two identical antennas in the far field, we can combine the above equations as

$$\tilde{V}_L(s) = \frac{e^{-\gamma r}}{r}\,\frac{s\,Z_o}{2\pi\,c\,R}\,\tilde{\vec{h}}_t(\vec{1}_i,s)\bullet\tilde{\vec{h}}_t(-\vec{1}_r,s)\,\tilde{V}_t(s) \tag{A.2}$$

where $\vec{1}_i = -\vec{1}_r$. It is perhaps more helpful to express the above equation in terms of the source voltage one would measure with a matched (50 Ω) scope. Thus, we can relate V_t to V_m, the measured scope output, as

$$\tilde{V}_t(s) = \tilde{V}_m(s)\,\frac{2\tilde{Z}_{in}(s)}{\tilde{Z}_{in}(s)+R} \tag{A.3}$$

So the calibration equation becomes

$$\tilde{V}_L(s) = \frac{e^{-\gamma r}}{r}\,\frac{s}{2\pi\,c}\,\frac{Z_o}{R}\,\frac{2\tilde{Z}_{in}(s)}{\tilde{Z}_{in}(s)+R}\,\tilde{\vec{h}}_t(\vec{1}_i,s)\bullet\tilde{\vec{h}}_t(\vec{1}_i,s)\,\tilde{V}_m(s) \tag{A.4}$$

One can then normalize the above equation as

$$\begin{aligned}
\tilde{V}_L(s) &= \frac{e^{-\gamma r}}{r}\,\frac{s}{2\pi\,c}\,\tilde{\vec{h}}_n(\vec{1}_i,s)\bullet\tilde{\vec{h}}_n(\vec{1}_i,s)\,\tilde{V}_m(s) \\
\tilde{\vec{h}}_n(\vec{1}_i,s) &= \sqrt{\frac{Z_o}{R}\,\frac{2\tilde{Z}_{in}(s)}{\tilde{Z}_{in}(s)+R}}\;\tilde{\vec{h}}_t(\vec{1}_i,s)
\end{aligned} \tag{A.5}$$

This is analogous to (6.4) and (7.4). However, it is more general in the sense that there are fewer restrictions on the various impedances.

It is important to distinguish between the results in this appendix, and the results in the earlier section of the paper. The $h_t(t)$ used here is not the same as the $h(t)$ used throughout the earlier part of this paper. So we do not expect the factors in front of equation (A.4) to be the same as those in front of (4.1) or (8.1). Recall that $h(t)$ as defined here always has an impulse area that is equal to half the dipole moment in the aperture, as shown in (2.4). That is not the case for $h_t(t)$ as used in this appendix. So, while the expressions developed in this appendix are rigorously correct, they are in a form that can be more clumsy to use in practice. Recall that whenever we make a measurement of an antenna's impulse response, we always must check to see that the impulse area is consistent with half of the dipole moment in the aperture. It is not clear how to do that with the expressions in this appendix.

In any case, it should be clear that when $h_N(t)$ is defined by equation (8.2), then there is no approximation or ambiguity in its definition. There is an approximation required to relate $h_N(t)$ to $h(t)$, but there appears to be no way to avoid that. The only reason one needs to relate $h_N(t)$ to $h(t)$ is as a check on the validity of a measurement using the line dipole method. If an antenna has a complex input impedance, then there can *never* be a simple relationship between the two. But if an antenna has an *approximately* real input impedance at early time, then this is a very useful check on the validity of the measurement.

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